

# Untagged $\bar{B} \rightarrow X_{s+d} \gamma$ CP asymmetry as a probe for new physics

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## Abstract

The direct CP asymmetry in the untagged inclusive channel  $\bar{B} \rightarrow X_{s+d} \gamma$  provides a strict test of the standard model. It has been shown beyond the partonic level that this asymmetry is negligibly small thanks to U-spin relations and to the unitarity of the CKM matrix. In the present paper we investigate this relation beyond the SM; in particular, we analyse to which extent deviations from this prediction are possible in supersymmetric scenarios. We analyse the minimal flavour violation scenario, including  $\tan \beta$ -enhanced terms and using the complete two-loop renormalization group running. Our analysis fully takes into account also the EDM constraints on the supersymmetric phases. We investigate possible correlations between the tagged and the untagged CP asymmetries and the indirect sensitivity of the latter to the  $\bar{B} \rightarrow X_d \gamma$  CP asymmetry.

Furthermore, we derive general model-independent formulae for the branching ratios and CP asymmetries for the inclusive  $\bar{B} \rightarrow X_d \gamma$  and  $\bar{B} \rightarrow X_s \gamma$  modes, and update the corresponding SM predictions. We obtain:

$$\begin{aligned} \mathcal{B}[\bar{B} \rightarrow X_s \gamma] &= (3.61 \pm_{-0.40}^{+0.24} \Big|_{m_c/m_b} \pm 0.02_{\text{CKM}} \pm 0.24_{\text{param.}} \pm 0.14_{\text{scale}}) \times 10^{-4}, \\ A_{\text{CP}}[\bar{B} \rightarrow X_s \gamma] &= (0.42 \pm_{-0.08}^{+0.08} \Big|_{m_c/m_b} \pm 0.03_{\text{CKM}} \pm_{-0.08}^{+0.15} \Big|_{\text{scale}}) \%. \end{aligned}$$

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# 1 Introduction

The CKM prescription of CP violation with one single phase is very predictive. It was proposed already in 1973 [1], before the experimental confirmation of the existence of the second family. Before the start of the  $B$  factories, the neutral kaon system was the only environment where CP violation had been observed. It has been difficult to decide if the CKM description of the standard model (SM) really accounted quantitatively for the CP violation observed in the kaon system, because of the large theoretical uncertainties due to long-range strong interactions. The rich data sets from the  $B$ -factories now allow for an independent and really quantitative test of the CKM-induced CP violating effects in several independent channels. Within the golden  $B$  mode  $B_d \rightarrow J/\psi K_S$  the CKM prescription of CP violation has already passed its first precision test; in fact, the measured CP violation is well in agreement with the CKM prediction [2, 3].

Nevertheless, there is still room for non-standard CP phases. An additional experimental test of the CKM mechanism is provided by the mode  $B_d \rightarrow \Phi K_S$ . This mode is induced at the loop level only and, therefore, it is much more sensitive to possible additional sources of CP violation than the tree-level-induced decay  $B_d \rightarrow J/\psi K_S$ . However, the poor statistics does not allow to draw final conclusions yet [4, 5]. Direct CP asymmetries in loop-induced  $\Delta F = 1$  modes allow for additional precision tests of the mechanism of CP violation. Currently, these decays are less probed than  $\Delta F = 2$  transitions. However, very precise measurements of direct CP asymmetries in inclusive rare  $B$  decays, such as  $b \rightarrow s$  or  $b \rightarrow d$  transitions, will be possible in the near future and they are the focus of the present paper.

Within the SM the direct CP asymmetry in the inclusive decay  $\bar{B} \rightarrow X_s \gamma$  is expected to be below 1%. This is a consequence of three suppression factors: (i) Direct CP violation requires at least two interfering contributions to the decay rate with different strong and weak phases; thus, an  $\alpha_s$  factor is needed in order to generate a strong phase. (ii) This interference receives a CKM suppression of order  $\lambda^2$ . (iii) There is a GIM suppression of order  $(m_c/m_b)^2$  reflecting the fact that, in the limit  $m_c = m_u$ , any CP asymmetry in the SM vanishes. It will be rather difficult to make an inclusive measurement of the CP asymmetry in the  $b \rightarrow d$  channel. However, based on CKM unitarity, one can derive a U-spin relation between the direct CP asymmetries in the  $b \rightarrow d$  and the  $b \rightarrow s$  channel [6]. U-spin breaking effects are estimated to be negligibly small. This finally leads to the SM zero-prediction for the CP asymmetry in the untagged mode  $\bar{B} \rightarrow X_{s+d} \gamma$  [7, 8]. This zero prediction provides a very clean test, whether new CP phases are active or not. Any significant deviation from this prediction would be a direct hint of non-CKM contributions to CP violation.

In the present paper we analyse this relation within various general scenarios beyond the SM. In [9] the untagged CP asymmetry was already considered in a specific model with vector quarks. We will first focus on supersymmetric models with minimal flavour violation and then consider a very general parameterization of new physics contributions. The first part of our analysis is, thus, based on the consistent definition of minimal flavour violation recently presented in [10] in which all flavour and CP-violating interactions originate from the Yukawa couplings. This constraint is introduced using an effective field approach supplemented by a symmetry concept and can be shown to be renormalization-group invariant. We

shall also extend consistently this definition by introducing flavour-blind phases. We realize the minimal flavour violating scenario in terms of a flavour-blind minimal supersymmetric standard model (MSSM) in which all the soft breaking terms are generated at the GUT scale. The case of general flavour violation is analysed in a model-independent framework. Taking into account the available experimental bounds on the  $\bar{B} \rightarrow X_s \gamma$  branching ratio and on the corresponding direct CP asymmetry, we analyse possible correlations between the tagged and untagged measurements and their indirect sensitivity to the asymmetry in the  $\bar{B} \rightarrow X_d \gamma$ .

We also derive general model-independent formulae for the branching ratios and the direct CP asymmetries for the two inclusive modes  $\bar{B} \rightarrow X_s \gamma$  and  $\bar{B} \rightarrow X_d \gamma$ , which also allow us to update the corresponding SM predictions.

Let us summarize the experimental situation and prospects. The present experimental accuracy on the inclusive decay  $\bar{B} \rightarrow X_s \gamma$  already reached the 10% level, as reflected in the world average of the present measurements [11–16]:

$$\mathcal{B}[\bar{B} \rightarrow X_s \gamma] = (3.34 \pm 0.38) \times 10^{-4}. \quad (1)$$

In the near future, more precise data on this mode are expected from the  $B$ -factories. In particular, the direct CP asymmetries are now within experimental reach. The first measurement of this asymmetry was presented by CLEO [17]; it is actually a weighted sum over the  $b \rightarrow s$  and  $b \rightarrow d$  channels:  $A_{\text{CP}} = 0.965 A_{\text{CP}}[\bar{B} \rightarrow X_s \gamma] + 0.02 A_{\text{CP}}[\bar{B} \rightarrow X_d \gamma]$ , yielding

$$A_{\text{CP}}[\bar{B} \rightarrow X_s \gamma] = (-0.079 \pm 0.108 \pm 0.022) \times (1.0 \pm 0.030). \quad (2)$$

The first error is statistical, the second and third errors are additive and multiplicative systematics respectively. This measurement is based on  $10^7 B\bar{B}$  events (on resonance); it uses fully inclusive and semi-inclusive techniques and implies  $-0.27 < A_{\text{CP}}[\bar{B} \rightarrow X_s \gamma] < +0.10$  at 90% confidence level. The recent Belle measurement [18] uses semi-inclusive techniques; it is based on  $15 \times 10^7 B\bar{B}$  events (on resonance) and leads to

$$A_{\text{CP}}[\bar{B} \rightarrow X_s \gamma] = -0.004 \pm 0.051 \pm 0.038, \quad (3)$$

where the first error is statistical and the second systematic. This corresponds to  $-0.107 < A_{\text{CP}} < 0.099$  at 90% confidence level. Very large effects are thus already experimentally excluded. Note that the same conclusion can be deduced from the measurements of the CP asymmetry in the exclusive  $B \rightarrow K^* \gamma$  modes. The world average includes CLEO, Babar and Belle measurements and reads [19].

$$A_{\text{CP}}[B \rightarrow K^* \gamma] = -0.005 \pm 0.037. \quad (4)$$

We stress that the application of quark–hadron duality is, in general, problematic within a semi-inclusive measurement of CP-violating effects, if only 50% or 70% of the total exclusive modes are detected. In fact, the strong rescattering phases responsible for the presence of CP violation can be different for each exclusive channel. It is impossible to reliably quantify the resulting systematic uncertainty without a detailed study of the individual modes and

of their direct CP asymmetries. Therefore, a fully inclusive measurement of the *untagged* direct CP asymmetry, the observable on which the present paper focuses, is favoured. Such a measurement is possible because the experimental efficiencies within the inclusive  $b \rightarrow s$  and  $b \rightarrow d$  modes are expected to be equal. Recent analyses of the future experimental accuracy [20] expect a total integrated luminosity of about  $1ab^{-1}$ , by the end of BaBar and Belle; this translates into an experimental error on the CP asymmetries of order 3%<sup>2</sup>. The potential of the so-called Super- $B$ -factories with an integrated luminosity of about  $50ab^{-1}$  would even lead to an experimental uncertainty of about 0.5% [20].

The plan of this paper is as follows: in the next section we derive in detail the improved model-independent formulae for the branching ratios and the direct tagged CP asymmetries for the two inclusive modes  $\bar{B} \rightarrow X_s \gamma$  and  $\bar{B} \rightarrow X_d \gamma$ . Here we also present our new SM predictions there. In Section 3 we discuss the untagged CP asymmetry within the SM. In Section 4 we present our analysis within the minimal flavour violation scenario. Finally, Section 5 contains the corresponding model-independent analysis – followed by our conclusions.

## 2 $B \rightarrow X_s, X_d \gamma$ decays beyond the SM: branching ratios and CP asymmetries

In this section we derive general model-independent formulae for the branching ratios and the direct tagged CP asymmetries for the inclusive  $\bar{B} \rightarrow X_{s,d} \gamma$  modes. Our main aim is to present explicit numerical expressions for these observables as functions of Wilson coefficients and CKM angles. The extraction of the latter from experimental data depends critically on the assumptions about the presence and the structure of new physics contributions to observables such as  $\Delta M_{B_d}$ ,  $\Delta M_{B_s}$ ,  $\epsilon_K$ ,  $a_{\psi K_s}$ . Therefore, the numerical expressions that we will present below in Eqs. (42) and (43) will be very useful in phenomenological analyses.

For this purpose we generalize the SM results at the NLL level given in Ref. [21]<sup>3</sup> in order to accommodate new physics models with new CP-violating phases as well as implement several improvements. We also update the corresponding SM predictions.

Let us start with the generalization of the NLL formulae and also the discussion of the input parameters:

- The general effective hamiltonian that governs the inclusive  $\bar{B} \rightarrow X_q \gamma$  decays ( $q = d, s$ ) in the SM is

$$H_{\text{eff}}(b \rightarrow q \gamma) = -\frac{4G_F}{\sqrt{2}} V_{tb} V_{tq}^* \left( \sum_{i=1}^8 C_i(\mu) \cdot O_i(\mu) + \epsilon_q \sum_{i=1}^2 C_i(\mu) \cdot (O_i(\mu) - O_i^u(\mu)) \right), \quad (5)$$

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<sup>2</sup>For semi-inclusive measurements the experimental error is expected to be even smaller. However, in this case, the additional systematic uncertainties discussed above must be taken into account.

<sup>3</sup>Reference [21] presents a detailed discussion of the NLL QCD formulae, which are based on the original NLL QCD calculations presented in [22–24] and on independent checks of these calculations [25–27].

where  $\epsilon_q = (V_{ub}V_{uq}^*)/(V_{tb}V_{tq}^*)$  and the operators are:

$$\begin{aligned}
O_1^u &= (\bar{q}_L \gamma_\mu T^a u_L)(\bar{u}_L \gamma^\mu T^a b_L), & O_2^u &= (\bar{q}_L \gamma_\mu u_L)(\bar{u}_L \gamma^\mu b_L), \\
O_1 &= (\bar{q}_L \gamma_\mu T^a c_L)(\bar{c}_L \gamma^\mu T^a b_L), & O_2 &= (\bar{q}_L \gamma_\mu c_L)(\bar{c}_L \gamma^\mu b_L) \\
O_3 &= (\bar{q}_L \gamma_\mu b_L) \sum_{q'} (\bar{q}'_L \gamma^\mu q'_L), & O_4 &= (\bar{q}_L \gamma_\mu T^a b_L) \sum_{q'} (\bar{q}'_L \gamma^\mu T^a q'_L), \\
O_5 &= (\bar{q}_L \gamma_\mu \gamma_\nu \gamma_\rho b_L) \sum_{q'} (\bar{q}'_L \gamma^\mu \gamma^\nu \gamma^\rho q'_L), & O_6 &= (\bar{q}_L \gamma_\mu \gamma_\nu \gamma_\rho T^a b_L) \sum_{q'} (\bar{q}'_L \gamma^\mu \gamma^\nu \gamma^\rho T^a q'_L), \\
O_7 &= \frac{e}{16\pi^2} m_b(\mu) (\bar{q}_L \sigma_{\mu\nu} b_R) F^{\mu\nu}, & O_8 &= \frac{g_s}{16\pi^2} m_b(\mu) (\bar{q}_L T^a \sigma_{\mu\nu} b_R) G^{a\mu\nu}.
\end{aligned} \tag{6}$$

We assume, within our model-independent analysis, that the dominant new physics effects only modify the Wilson coefficients of the dipole operators  $O_7$  and  $O_8$  and also introduce contributions proportional to the corresponding operators with opposite chirality:

$$O_7^R = \frac{e}{16\pi^2} m_b(\mu) (\bar{q}_R \sigma_{\mu\nu} b_L) F^{\mu\nu}, \quad O_8^R = \frac{g_s}{16\pi^2} m_b(\mu) (\bar{q}_R T^a \sigma_{\mu\nu} b_L) G^{a\mu\nu}. \tag{7}$$

- The branching ratio for  $\bar{B} \rightarrow X_q \gamma$  can be parameterized as

$$\mathcal{B}[\bar{B} \rightarrow X_q \gamma]_{E_\gamma > E_0}^{\text{subtracted } \psi, \psi'} = \mathcal{B}[\bar{B} \rightarrow X_c e \bar{\nu}]_{\text{exp}} \frac{6\alpha_{\text{em}}}{\pi C} \left| \frac{V_{tq}^* V_{tb}}{V_{cb}} \right|^2 [P(E_0) + N(E_0)] \tag{8}$$

$$= \mathcal{N} \left| \frac{V_{tq}^* V_{tb}}{V_{cb}} \right|^2 [P(E_0) + N(E_0)], \tag{9}$$

where  $P(E_0)$  and  $N(E_0)$  denote the perturbative and the non-perturbative contributions respectively.

- The pre-factor  $C$  in Eq. (9) is given by

$$C = \left| \frac{V_{ub}}{V_{cb}} \right|^2 \frac{\Gamma[\bar{B} \rightarrow X_c e \bar{\nu}]}{\Gamma[\bar{B} \rightarrow X_u e \bar{\nu}]}. \tag{10}$$

In Ref. [21], the authors present a detailed determination of  $C$  employing the  $\Upsilon$  expansion [28]. The uncertainties on  $C$  are dominated by the errors on the heavy quark effective theory parameter  $\lambda_1$  and on the non-perturbative contribution to the  $\Upsilon$  mass,  $\Delta = m_\Upsilon/2 - m_b^{1S}$ , where  $m_b^{1S}$  is defined as half of the perturbative contribution to the  $\Upsilon$  mass. Using  $\lambda_1 = (-0.27 \pm 0.10 \pm 0.04) \text{ GeV}^2$  [28] and  $m_b^{1S} = (4.69 \pm 0.03) \text{ GeV}$  [29], from which follows  $\Delta^2/(m_\Upsilon/2) = (0.04 \pm 0.03) \text{ GeV}$ , the authors of ref. [21] obtain  $C = 0.575 (1 \pm 0.01_{\text{pert}} \pm 0.02_{\lambda_1} \pm 0.02_\Delta) = 0.575 (1 \pm 0.03)$ . In this analysis we prefer to increase the controversially small error on  $m_b^{1S}$  given in Ref. [29] (see the discussion in Sec. 7 of Ref. [30]). Moreover we increase the error on  $\lambda_1$  in order to include the effects of the unknown  $\Lambda_{\text{QCD}}^3/m_D^2$  corrections to the  $m_c^{\text{pole}}/m_b^{\text{pole}}$  ratio. Our more conservative error analysis leads us to  $C = 0.575 (1 \pm 0.01_{\text{pert}} \pm 0.04_{\lambda_1} \pm 0.04_\Delta) = 0.575 (1 \pm 0.06)$ . Using  $\mathcal{B}[\bar{B} \rightarrow X_c e \bar{\nu}]_{\text{exp}} = 0.1074 \pm 0.0024$  and  $\alpha_{\text{em}} = 1/137.036$ , we finally obtain

$$\mathcal{N} = 2.567 (1 \pm 0.064) \times 10^{-3}. \tag{11}$$

- In this paper we use the Wolfenstein parameters of the CKM matrix according to the fit presented in Ref. [31] ( $\lambda = 0.2240 \pm 0.0036$ ,  $A = 0.83 \pm 0.02$ ,  $\bar{\rho} = 0.162 \pm 0.046$ ,  $\bar{\eta} = 0.347 \pm 0.027$ ) and obtain

$$\left| \frac{V_{ts}^* V_{tb}}{V_{cb}} \right|^2 = 0.9648 (1 \pm 0.005), \quad (12)$$

$$\left| \frac{V_{td}^* V_{tb}}{V_{cb}} \right|^2 = 0.0412 (1 \pm 0.10) \quad (13)$$

$$\frac{V_{us}^* V_{ub}}{V_{ts}^* V_{tb}} = \epsilon_s = (-0.0088 \pm 0.0024) + i (0.0180 \pm 0.0015), \quad (14)$$

$$\frac{V_{ud}^* V_{ub}}{V_{td}^* V_{tb}} = \epsilon_d = (0.019 \pm 0.046) - i (0.422 \pm 0.046). \quad (15)$$

From these values it is obvious that CKM uncertainties are completely negligible in  $b \rightarrow s\gamma$  transitions but play an important role in  $b \rightarrow d\gamma$  ones.

- The non-perturbative contribution  $N(E_0)$  in Eq. (9) is not sensitive to new physics and we will use the numerical estimate of Ref. [21]:

$$N(E_0) = 0.0036 \pm 0.0006. \quad (16)$$

- The perturbative contribution  $P(E_0)$  is defined by [21]

$$\frac{\Gamma[b \rightarrow X_q \gamma]_{E_\gamma > E_0}}{|V_{cb}/V_{ub}|^2 \Gamma[b \rightarrow X_u e \bar{\nu}]} = \left| \frac{V_{tq}^* V_{tb}}{V_{cb}} \right|^2 \frac{6\alpha_{\text{em}}}{\pi} P(E_0). \quad (17)$$

This contribution can be parameterized in the following way:

$$P(E_0) = \left| K_c + \left( 1 + \frac{\alpha_s(\mu_0)}{\pi} \log \frac{\mu_0^2}{m_t^2} \right) r(\mu_0) K_t + \varepsilon_{\text{ew}} \right|^2 + B(E_0), \quad (18)$$

where  $K_t$  is the top contribution to the  $b \rightarrow q\gamma$  amplitude,  $K_c$  contains the remaining contributions (including those from the operators  $O_i^u$ ) and  $\varepsilon_{\text{ew}}$  are the electroweak corrections;  $r(\mu_0)$  is the ratio of the  $\overline{\text{MS}}$  running mass of the bottom quark ( $m_b^{\overline{\text{MS}}}(\mu_0)$ ) to the 1S mass ( $m_b^{1S}$ ). The expression  $r(\mu_0)$  can be found in the appendix;  $\mu_0$  denotes the matching scale, typically  $m_W$ . Finally, the function  $B(E_0)$  contains the bremsstrahlung effects coming from the process  $b \rightarrow q\gamma g$ .

- The contribution  $K_c$  is practically insensitive to new physics because the dominant new physics contributions change the dipole operator contributions only. The explicit form of  $K_c$  we use here is slightly different from the one presented in Ref. [21]; in fact, the inclusion of all two-loop matrix elements of the 4-quark operators, given in [32], requires a small modification of the term proportional to  $\epsilon_q$ :

$$K_c = K_c^{(0)} + K_c^{(11)} + i K_c^{(12)} + \left( K_c^{(13)} + i K_c^{(14)} \right) \epsilon_q, \quad (19)$$

where the quantities  $K_c^{ij}$  are real and their explicit expressions are given in the appendix;  $K_c^{(0)}$  and the sum of the  $K_c^{(1j)}$  are the LL and NLL contributions, respectively. Only the terms  $K_c^{(13)}$  and  $K_c^{(14)}$  differ from Ref. [21]. This modification is only relevant to the  $b \rightarrow d$  mode and is therefore not included in the NLL formulae of [32].

- The top contribution  $K_t$ , however, must be generalized to include new physics with generic CP-violating phases. As already explained in Ref. [21] its expression coincides with Eq. (5.1) of Ref. [21] after the substitutions:

$$C_{7,8}^{(0)\text{SM}}(\mu_0) \rightarrow C_{7,8}^{(0)\text{SM}}(\mu_0) + C_{7,8}^{(0)\text{NP}}(\mu_0) = C_{7,8}^{(0)\text{tot}}(\mu_0) \quad (20)$$

where

$$C_7^{(0)\text{SM}}(\mu_0) = -\frac{1}{2}A_0^t(x_t) - \frac{23}{36} \quad \text{and} \quad C_8^{(0)\text{SM}}(\mu_0) = -\frac{1}{2}F_0^t(x_t) - \frac{1}{3}. \quad (21)$$

with  $x_t = (m_t(\mu_0)/m_W)^2$ . The functions  $A_0^t$  and  $F_0^t$  are explicitly defined in [21]. Note that the terms proportional to  $\log(\mu_0/m_t)$  have to be treated carefully [21].

It is convenient to parameterize the result as follows:

$$K_t = K_t^{(0)} + K_t^{(1)} + iK_t^{(1)i}, \quad (22)$$

where  $K_t^{(0)}$  is the total LL contribution and  $K_t^{(1)} + iK_t^{(1)i}$  is the NLL one. The explicit  $i$  factor is the only strong phase present in the whole top contribution. These contributions can be further decomposed in terms of the total Wilson coefficients of the operators  $O_{7,8}$  evaluated at the matching scale  $\mu_0$ :

$$K_t^{(0)} = K_t^{(01)} + K_t^{(02)} C_7^{(0)\text{tot}}(\mu_0) + K_t^{(03)} C_8^{(0)\text{tot}}(\mu_0), \quad (23)$$

$$K_t^{(1)} = K_t^{(11)} + K_t^{(12)} C_7^{(0)\text{tot}}(\mu_0) + K_t^{(13)} C_8^{(0)\text{tot}}(\mu_0), \quad (24)$$

$$K_t^{(1)i} = K_t^{(14)} + K_t^{(15)} C_8^{(0)\text{tot}}(\mu_0). \quad (25)$$

We emphasize again that all the  $K_t^{(ij)}$  are real, and we list them in the appendix. The Wilson coefficients in the SM are numerically given by

$$C_7^{(0)\text{SM}}(m_t) = -0.189 \quad \text{and} \quad C_8^{(0)\text{SM}}(m_t) = -0.095, \quad (26)$$

where we used  $m_t^{\overline{\text{MS}}}(m_t^{\overline{\text{MS}}}) = (165 \pm 5) \text{ GeV}$ .

In principle, one could add the NLL new physics contributions to the NLL Wilson coefficients by replacing  $C_{7,8}^{(1)\text{SM}}(\mu_0) \rightarrow C_{7,8}^{(1)\text{SM}}(\mu_0) + C_{7,8}^{(1)\text{NP}}(\mu_0)$ . However, in the numerical analysis, we set  $C_{7,8}^{(1)\text{NP}}(\mu_0) = 0$  and, thus, effectively describe all new physics effects by the LL Wilson coefficients  $C_{7,8}^{\text{NP}}$ .

- The electroweak corrections can be written as:

$$\varepsilon_{\text{ew}} = \varepsilon_{\text{ew}}^{\text{SM}} + C_7^{(0)\text{NP}}(\mu_0) \varepsilon_{\text{ew}}^{(11)} + C_8^{(0)\text{NP}}(\mu_0) \varepsilon_{\text{ew}}^{(12)}, \quad (27)$$

where  $\varepsilon_{\text{ew}}^{\text{SM}} = 0.0071$  is the SM contribution [21]; the formulae for  $\varepsilon_{\text{ew}}^{(ij)}$  can be found in the appendix.

- The function  $B(E_0)$  contains the bremsstrahlung effects coming from  $b \rightarrow s\gamma g$ . Allowing for complex Wilson coefficients we get:

$$\begin{aligned}
B(E_0) = & \frac{\alpha_s(\mu_b)}{\pi} \text{Re} \left\{ \sum_{i \leq j=7}^8 C_i^{(0)\text{eff}}(\mu_b) C_j^{(0)\text{eff}*}(\mu_b) \phi_{ij}(\delta, z) \right. \\
& + \sum_{i \leq j=1}^2 C_i^{(0)\text{eff}}(\mu_b) C_j^{(0)\text{eff}*}(\mu_b) \left[ |1 + \epsilon_q|^2 \phi_{ij}(\delta, z) + |\epsilon_q|^2 \phi_{ij}(\delta, 0) \right] \\
& \left. + \sum_{\substack{i=1,2 \\ j=7,8}} C_i^{(0)\text{eff}}(\mu_b) C_j^{(0)\text{eff}*}(\mu_b) \left[ (1 + \epsilon_q) \phi_{ij}(\delta, z) - \epsilon_q \phi_{ij}(\delta, 0) \right] \right\}, \quad (28)
\end{aligned}$$

where  $\delta = 1 - 2E_0/m_b$ ,  $z = m_c^2/m_b^2$ , and the  $\phi_{ij}(\delta, z)$  are given in the appendix. The Wilson coefficients  $C_j^{(0)\text{eff}}$  are given in Eq. (E.9) of Ref. [21]. The inclusion of new physics contributions to the Wilson coefficients  $C_{7,8}^{(0)\text{eff}}$  follows from the substitutions Eq. (20). Note that in contrast to the virtual contribution we have neglected the contribution of the QCD penguin operators,  $O_{3-6}$ , within the bremsstrahlung contribution whose impact on the branching ratio is at the 0.1% level [21]. Moreover, there are additional terms coming from the interference  $O_{1,2}^u - O_{1,2}$  that have been pointed out for the first time in Ref. [33]. They are only relevant to the CP asymmetry within the  $b \rightarrow d$  sector. However, we neglect them in this analysis because their contribution is below the per-cent level.

- It is straightforward to generalize our equations to include new physics contributions to the dipole operators with opposite chirality given in Eq. (7). Note that  $O_{7,8}^R$  do not interfere with  $O_{1-8}$  and  $O_{1,2}^u$ , hence terms linear in  $C_{7R}^{(0)}(\mu_0)$  and  $C_{8R}^{(0)}(\mu_0)$  are absent. The quadratic terms can be easily included using the following prescription:

$$\begin{aligned}
C_i^{(0)\text{tot}}(\mu_0) C_j^{(0)\text{tot}*}(\mu_0) \longrightarrow & C_i^{(0)\text{tot}}(\mu_0) C_j^{(0)\text{tot}*}(\mu_0) \\
& + C_{iR}^{(0)\text{tot}}(\mu_0) C_{jR}^{(0)\text{tot}*}(\mu_0) \quad (i, j = 7, 8). \quad (29)
\end{aligned}$$

- Let us briefly comment on the choices of the scales  $\mu_b$  and  $\mu_0$ . We take  $\mu_b = m_b^{1S}$  and vary it by a factor of 2. Following Ref. [21] we use  $\mu_0 = m_W$  in  $K_c$  and  $B(E_0)$ , while  $\mu_0 = m_t^{\overline{\text{MS}}}(m_t^{\overline{\text{MS}}})$  in  $K_t$  and  $r(\mu_0)$ . For the new physics contributions to the Wilson coefficients we also use  $\mu_0 = m_t^{\overline{\text{MS}}}(m_t^{\overline{\text{MS}}})$ .
- The issue regarding the choice of the charm mass definition in the matrix element of  $O_2$  deserves a discussion. In Ref. [21], it is argued that all the factors of  $m_c$  come from propagators corresponding to charm quarks that are off-shell by an amount  $\mu^2 \sim m_b^2$ . Therefore it seems more reasonable to use the  $\overline{\text{MS}}$  running charm mass at a scale  $\mu$  in the range  $(m_c, m_b)$ . Here and in the following the reference values of the charm and bottom masses are  $m_c = m_c^{\overline{\text{MS}}}(m_c^{\overline{\text{MS}}}) = (1.25 \pm 0.10) \text{ GeV}$  and  $m_b = m_b^{1S}$ . We first fix the central value of  $m_c = 1.25 \text{ GeV}$  and vary  $\mu$ ; then we add in quadrature the error on  $m_c$  ( $\delta_{m_c} = 8\%$ ). The resulting determination is:

$$\frac{m_c}{m_b} = 0.23 \pm 0.05. \quad (30)$$



The pole mass choice corresponds, on the other hand, to  $\frac{m_c}{m_b} = 0.29 \pm 0.02$ . Note that the question of whether to use the running or the pole mass is, strictly speaking, a NNLL issue. The most conservative position consists in accepting any value of  $m_c/m_b$  that is compatible with any of these two determinations:  $0.18 \leq m_c/m_b \leq 0.31$ . Taking into account all past NLL computations, we strongly believe that the central value  $m_c/m_b = 0.23$  represents the best possible choice, but we allow for a large asymmetric error that fully covers the above range (and that reminds us of this problem that can be solved only via a NNLL computation):

$$\frac{m_c}{m_b} = 0.23^{+0.08}_{-0.05}. \quad (31)$$

After this discussion of the necessary generalizations of the NLL formulae, we present our SM updates of the branching ratios, the CP asymmetries, and our formulae for the model-independent NLL analysis.

- We collect first the SM predictions for the branching ratios using Eq. (31) and two different energy cuts:  $E_0 = (1.6 \text{ GeV}, m_b/20)$ . There are four sources of uncertainties: the charm mass ( $\delta_{m_c/m_b}$ ), the CKM factors ( $\delta_{\text{CKM}}(s) = 0.5\%$ ,  $\delta_{\text{CKM}}(d) = 11\%$ ), the parametric uncertainty, including the one due to the overall normalization  $\mathcal{N}$ ,  $\alpha_s$  and  $m_t$  ( $\delta_{\text{param.}}$ ) and the perturbative scale uncertainty ( $\delta_{\text{scale}}$ ). Concerning the latter two errors, we follow the analysis of Ref. [21] and use  $\delta_{\text{param.}} = 6.4\%$  and  $\delta_{\text{scale}} = 4\%$ . For  $E_\gamma > 1.6 \text{ GeV}$  we get:

$$\mathcal{B}[\bar{B} \rightarrow X_s \gamma] = \left( 3.61 \begin{smallmatrix} +0.24 \\ -0.40 \end{smallmatrix} \left| \frac{m_c}{m_b} \right. \pm 0.02_{\text{CKM}} \pm 0.24_{\text{param.}} \pm 0.14_{\text{scale}} \right) \times 10^{-4}, \quad (32)$$

$$\mathcal{B}[\bar{B} \rightarrow X_d \gamma] = \left( 1.38 \begin{smallmatrix} +0.14 \\ -0.21 \end{smallmatrix} \left| \frac{m_c}{m_b} \right. \pm 0.15_{\text{CKM}} \pm 0.09_{\text{param.}} \pm 0.05_{\text{scale}} \right) \times 10^{-5}, \quad (33)$$

$$\frac{\mathcal{B}[\bar{B} \rightarrow X_d \gamma]}{\mathcal{B}[\bar{B} \rightarrow X_s \gamma]} = \left( 3.82 \begin{smallmatrix} +0.11 \\ -0.18 \end{smallmatrix} \left| \frac{m_c}{m_b} \right. \pm 0.42_{\text{CKM}} \pm 0.08_{\text{param.}} \pm 0.15_{\text{scale}} \right) \times 10^{-2}. \quad (34)$$

For  $E_\gamma > m_b/20$  we get:

$$\mathcal{B}[\bar{B} \rightarrow X_s \gamma] = \left( 3.79 \begin{smallmatrix} +0.26 \\ -0.44 \end{smallmatrix} \left| \frac{m_c}{m_b} \right. \pm 0.02_{\text{CKM}} \pm 0.25_{\text{param.}} \pm 0.15_{\text{scale}} \right) \times 10^{-4}, \quad (35)$$

$$\mathcal{B}[\bar{B} \rightarrow X_d \gamma] = \left( 1.46 \begin{smallmatrix} +0.15 \\ -0.23 \end{smallmatrix} \left| \frac{m_c}{m_b} \right. \pm 0.16_{\text{CKM}} \pm 0.10_{\text{param.}} \pm 0.06_{\text{scale}} \right) \times 10^{-5}, \quad (36)$$

$$\frac{\mathcal{B}[\bar{B} \rightarrow X_d \gamma]}{\mathcal{B}[\bar{B} \rightarrow X_s \gamma]} = \left( 3.86 \begin{smallmatrix} +0.11 \\ -0.18 \end{smallmatrix} \left| \frac{m_c}{m_b} \right. \pm 0.43_{\text{CKM}} \pm 0.09_{\text{param.}} \pm 0.15_{\text{scale}} \right) \times 10^{-2}. \quad (37)$$

Note that the errors on the ratio  $R_{ds} = \mathcal{B}[\bar{B} \rightarrow X_d \gamma]/\mathcal{B}[\bar{B} \rightarrow X_s \gamma]$  are dominated by CKM uncertainties. We remind the reader that, on top of the mentioned sources of error, the  $B \rightarrow X_d \gamma$  mode is affected by the presence of non-perturbative  $u$ -quark loops whose effect is expected to be at most around 10% (see section VI.B of Ref. [34] for a more detailed discussion).

- The direct CP asymmetries in  $\bar{B} \rightarrow X_q \gamma$  can immediately be extracted from the explicit expression Eq. (9) for the branching ratio:

$$A_{\text{CP}}^{b \rightarrow q\gamma} \equiv \frac{\Gamma[\bar{B} \rightarrow X_q \gamma] - \Gamma[B \rightarrow X_{\bar{q}} \gamma]}{\Gamma[\bar{B} \rightarrow X_q \gamma] + \Gamma[B \rightarrow X_{\bar{q}} \gamma]} \quad (38)$$

$$= \frac{\Delta\Gamma_{q\gamma} + \Delta\Gamma_{qg\gamma}}{P(E_0)}, \quad (39)$$

where  $P(E_0)$  is defined in Eq. (18) and  $\Delta\Gamma_{q\gamma, qg\gamma}$  are the contributions corresponding to the two terms in  $P(E_0)$  (virtual corrections and bremsstrahlung) and are given in the appendix. The rationale for normalizing the CP asymmetry, using the complete NLL expression for the CP-averaged branching ratio, relies on the observation that  $\Gamma[\bar{B} \rightarrow X_q \gamma]$  and  $\Gamma[B \rightarrow X_{\bar{q}} \gamma]$  are distinct observables: we are, therefore, allowed to compute them independently to the best of our knowledge.

The SM predictions are essentially independent of the photon energy cut-off ( $E_0$ ) and read (for  $E_0 = 1.6 \text{ GeV}$ ):

$$A_{\text{CP}}[\bar{B} \rightarrow X_s \gamma] = \left( 0.44 \begin{smallmatrix} +0.15 \\ -0.10 \end{smallmatrix} \Big|_{\frac{m_c}{m_b}} \pm 0.03_{\text{CKM}} \begin{smallmatrix} +0.19 \\ -0.09 \end{smallmatrix} \Big|_{\text{scale}} \right) \%, \quad (40)$$

$$A_{\text{CP}}[\bar{B} \rightarrow X_d \gamma] = \left( -10.2 \begin{smallmatrix} +2.4 \\ -3.7 \end{smallmatrix} \Big|_{\frac{m_c}{m_b}} \pm 1.0_{\text{CKM}} \begin{smallmatrix} +2.1 \\ -4.4 \end{smallmatrix} \Big|_{\text{scale}} \right) \%. \quad (41)$$

The additional parametric uncertainties are subdominant.

- Finally, we present our formulae for the branching ratios and CP asymmetries, in which the Wilson coefficients  $C_{7,8(R)}$  and all the CKM ratios are left unspecified:

$$\begin{aligned} \mathcal{B}[\bar{B} \rightarrow X_q \gamma] &= \frac{\mathcal{N}}{100} \left| \frac{V_{tq}^* V_{tb}}{V_{cb}} \right|^2 \left[ a + a_{77} (|R_7|^2 + |\tilde{R}_7|^2) + a_7^r \text{Re}(R_7) + a_7^i \text{Im}(R_7) \right. \\ &\quad + a_{88} (|R_8|^2 + |\tilde{R}_8|^2) + a_8^r \text{Re}(R_8) + a_8^i \text{Im}(R_8) + a_{\epsilon\epsilon} |\epsilon_q|^2 + a_\epsilon^r \text{Re}(\epsilon_q) + a_\epsilon^i \text{Im}(\epsilon_q) \\ &\quad + a_{87}^r \text{Re}(R_8 R_7^* + \tilde{R}_8 \tilde{R}_7^*) + a_{7\epsilon}^r \text{Re}(R_7 \epsilon_q^*) + a_{8\epsilon}^r \text{Re}(R_8 \epsilon_q^*) \\ &\quad \left. + a_{87}^i \text{Im}(R_8 R_7^* + \tilde{R}_8 \tilde{R}_7^*) + a_{7\epsilon}^i \text{Im}(R_7 \epsilon_q^*) + a_{8\epsilon}^i \text{Im}(R_8 \epsilon_q^*) \right], \end{aligned} \quad (42)$$

$$A_{\text{CP}}^{b \rightarrow q\gamma} = \frac{\text{Im} \left[ a_7^i R_7 + a_8^i R_8 + a_\epsilon^i \epsilon_q + a_{87}^i (R_8 R_7^* + \tilde{R}_8 \tilde{R}_7^*) + a_{7\epsilon}^i R_7 \epsilon_q^* + a_{8\epsilon}^i R_8 \epsilon_q^* \right]}{\frac{100}{\mathcal{N}} \left| \frac{V_{cb}}{V_{tq}^* V_{tb}} \right|^2 \frac{1}{2} (\mathcal{B}[\bar{B} \rightarrow X_q \gamma] + \mathcal{B}[B \rightarrow X_{\bar{q}} \gamma])}, \quad (43)$$

where

$$R_{7,8} = \frac{C_{7,8}^{(0)\text{tot}}(\mu_0)}{C_{7,8}^{(0)\text{SM}}(m_t)} \quad \text{and} \quad \tilde{R}_{7,8} = \frac{C_{7R,8R}^{(0)\text{NP}}(\mu_0)}{C_{7,8}^{(0)\text{SM}}(m_t)} \quad (44)$$

|                        | NLL     |         |          |         | LL     |
|------------------------|---------|---------|----------|---------|--------|
| $E_0$                  | 1.6 GeV |         | $m_b/20$ |         | -      |
| $m_c/m_b$              | 0.23    | 0.29    | 0.23     | 0.29    | -      |
| $a$                    | 7.8221  | 6.9120  | 8.1819   | 7.1714  | 7.9699 |
| $a_{77}$               | 0.8161  | 0.8161  | 0.8283   | 0.8283  | 0.9338 |
| $a_7^r$                | 4.8802  | 4.5689  | 4.9228   | 4.6035  | 5.3314 |
| $a_7^i$                | 0.3546  | 0.2167  | 0.3322   | 0.2029  | 0      |
| $a_{88}$               | 0.0197  | 0.0197  | 0.0986   | 0.0986  | 0.0066 |
| $a_8^r$                | 0.5680  | 0.5463  | 0.7810   | 0.7600  | 0.4498 |
| $a_8^i$                | -0.0987 | -0.1105 | -0.0963  | -0.1091 | 0      |
| $a_{\epsilon\epsilon}$ | 0.4384  | 0.3787  | 0.8598   | 0.7097  | 0      |
| $a_\epsilon^r$         | -1.6981 | -2.6679 | -1.3329  | -2.4935 | 0      |
| $a_\epsilon^i$         | 2.4997  | 2.8956  | 2.5274   | 2.9127  | 0      |
| $a_{87}^r$             | 0.1923  | 0.1923  | 0.2025   | 0.2025  | 0.1576 |
| $a_{87}^i$             | -0.0487 | -0.0487 | -0.0487  | -0.0487 | 0      |
| $a_{7\epsilon}^r$      | -0.7827 | -1.0940 | -0.8092  | -1.1285 | 0      |
| $a_{7\epsilon}^i$      | -0.9067 | -1.0447 | -0.9291  | -1.0585 | 0      |
| $a_{8\epsilon}^r$      | -0.0601 | -0.0819 | -0.0573  | -0.0783 | 0      |
| $a_{8\epsilon}^i$      | -0.0661 | -0.0779 | -0.0637  | -0.0765 | 0      |

Table 1: Numerical values of the coefficients introduced in Eqs. (42) and (43). We give the values corresponding to  $E_0 = (1.6 \text{ GeV}, m_b/20)$  and to  $m_c/m_b = (0.23, 0.29)$ . In the last column we give the values obtained at LL.

and the CP conjugate branching ratio,  $\mathcal{B}[B \rightarrow X_q \gamma]$ , can be obtained by Eq. (42) by replacing  $\text{Im}(\dots) \rightarrow -\text{Im}(\dots)$ . Explicit expressions for these coefficients in terms of the quantities introduced in Eqs. (19),(23)-(25),(27) can now be easily derived. Their numerical values are collected in Table 1.

### 3 Untagged $B \rightarrow X_{s+d} \gamma$ CP asymmetry

The unnormalized CP asymmetry for the sum of the partonic processes  $b \rightarrow (s+d)\gamma$  vanishes in the limit of  $m_d = m_s = 0$  as was first observed in Ref. [6]. This is still valid for the weaker condition  $m_d = m_s$ , which corresponds to the so-called U-spin limit. However, any CP violation in the SM has to be proportional to the determinant

$$\det [\mathcal{M}_U \mathcal{M}_D] = i J (m_u - m_c)(m_u - m_t)(m_c - m_t)(m_d - m_s)(m_d - m_b)(m_s - m_b), \quad (45)$$

where  $\mathcal{M}_{U/D}$  are the mass matrices for the up and down quarks and

$$J = \text{Im}[V_{ub}V_{cb}^*V_{cs}V_{us}^*] \quad (46)$$

is the Jarlskog parameter, which is the only fourth-order quantity in the SM invariant under rephasing of the quarks fields. If the down and the strange quark were degenerate, the SM would be completely CP-conserving, as can be seen from (45). Thus, the U-spin limit at the quark level does not make much sense with respect to CP asymmetries. However, one shall make use of this symmetry only with respect to the influence of the strong interactions on the hadronic matrix elements (in particular on the strong phases), while the down and strange quark masses are different. The unitarity of the CKM matrix implies

$$J = \text{Im}(\lambda_u^{(s)}\lambda_c^{(s)*}) = -\text{Im}(\lambda_u^{(d)}\lambda_c^{(d)*}), \quad (47)$$

where  $\lambda_q^{(q')} = V_{qb}V_{qq'}^*$ . As a consequence one finds in the U-spin limit for the hadronic matrix elements and for real Wilson coefficients the following relation for the rate asymmetries:

$$\Delta\Gamma(\bar{B} \rightarrow X_s\gamma) + \Delta\Gamma(\bar{B} \rightarrow X_d\gamma) = \Delta\Gamma_s + \Delta\Gamma_d = 0, \quad (48)$$

where  $\Delta\Gamma_q = \Delta\Gamma(\bar{B} \rightarrow X_q\gamma) = \Gamma(\bar{B} \rightarrow X_q\gamma) - \Gamma(B \rightarrow X_{\bar{q}}\gamma)$ .

U-spin breaking effects can be estimated within the heavy mass expansion even beyond the partonic level [7, 8]:

$$\Delta\Gamma(\bar{B} \rightarrow X_s\gamma) + \Delta\Gamma(\bar{B} \rightarrow X_d\gamma) = b_{\text{inc}} \Delta_{\text{inc}} \quad (49)$$

where the right-hand side is written as a product of a ‘relative U-spin breaking’  $b_{\text{inc}}$  and a ‘typical size’  $\Delta_{\text{inc}}$  of the CP violating rate difference. In this framework one relies on parton-hadron duality and one can compute the breaking of U-Spin by keeping a non-vanishing strange quark mass. A rough estimate of  $b_{\text{inc}}$  gives a value of the order of  $|b_{\text{inc}}| \sim m_s^2/m_b^2 \sim 5 \times 10^{-4}$ , while  $|\Delta_{\text{inc}}|$  is the average of the moduli of the two CP rate asymmetries. Thus, one arrives at the following estimate within the partonic contribution [7]:

$$|\Delta\mathcal{B}(B \rightarrow X_s\gamma) + \Delta\mathcal{B}(B \rightarrow X_d\gamma)| \sim 1 \cdot 10^{-9}. \quad (50)$$

Going beyond the leading partonic contribution within the heavy mass expansion, one has to check if the large suppression factor from the U-spin breaking,  $b_{\text{inc}}$ , is still effective in addition to the natural suppression factors already present in the higher order terms of the heavy mass expansion [8]. In the leading  $1/m_b^2$  corrections, the U-spin-breaking effects also induce an additional overall factor  $m_s^2/m_b^2$ . In the non-perturbative corrections from the charm quark loop, which scale with  $1/m_c^2$ , one finds again the same overall suppression factor, because the effective operators involved do not contain any information on the strange mass. Also the corresponding long-distance contributions from up-quark loops, which scale with  $\Lambda_{\text{QCD}}/m_b$ , follow the same pattern [34].

Thus, in the inclusive mode, the right-hand side in (50) can be computed in a model-independent way, with the help of the heavy mass expansion, and the U-spin breaking effects

can be estimated to be practically zero<sup>4</sup>. Therefore, the prediction (50) provides a very clean SM test, whether generic new CP phases are active or not. Any significant deviation from the estimate (50) would be a direct hint to non-CKM contributions to CP violation. This implies that any measurement of a non-zero untagged CP asymmetry is a direct signal for new physics beyond the SM.

A simple expression of the untagged CP asymmetry is given by:

$$A_{\text{CP}}(B \rightarrow X_{s+d}\gamma) = \frac{\Delta\Gamma_s + \Delta\Gamma_d}{\Sigma\Gamma_s + \Sigma\Gamma_d} = \frac{A_{\text{CP}}(B \rightarrow X_s\gamma) + R_{ds} A_{\text{CP}}(B \rightarrow X_d\gamma)}{1 + R_{ds}} \quad (51)$$

where  $\Sigma\Gamma_q = \Gamma(\bar{B} \rightarrow X_q\gamma) + \Gamma(B \rightarrow X_q\gamma)$  and  $R_{ds} = \Sigma\Gamma_d/\Sigma\Gamma_s$  ( $\Delta\Gamma_q$  is defined above).

From Eq. (51) again one easily derives that in the SM the untagged asymmetry  $A_{\text{CP}}(B \rightarrow X_{s+d}\gamma)$  vanishes identically: CKM unitarity and the reality of the Wilson coefficients in the SM imply  $\Delta\Gamma_s = c_s |V_{ts}| \text{Im}\epsilon_s$  and  $\Delta\Gamma_d = c_d |V_{td}| \text{Im}\epsilon_d$  where  $\epsilon_q = (V_{uq}^* V_{ub})/(V_{tq}^* V_{tb})$ . The U-spin symmetry for the hadronic matrix elements then implies  $c_s = c_d$  and one gets finally:

$$A_{\text{CP}}^{\text{SM}}(B \rightarrow X_{s/d}\gamma) \propto \text{Im}(\epsilon_s + |V_{td}/V_{ts}|^2 \epsilon_d) = 0. \quad (52)$$

We first note that recent experimental results from BABAR [36], put the following upper limit on the ratio of exclusive  $B$  decays  $B \rightarrow K^*\gamma$  and  $B \rightarrow \rho\gamma$

$$R(\rho\gamma/K^*\gamma) = \frac{\Gamma(B \rightarrow \rho\gamma)}{\Gamma(B \rightarrow K^*\gamma)} \leq 0.047 \text{ at } 90\% \text{ C.L.} \quad (53)$$

Assuming  $R_{ds} \sim R(\rho\gamma/K^*\gamma)$  one can conclude that all new physics models, in which the  $B \rightarrow X_s\gamma$  CP asymmetry is  $\sim 5\%$ , also predict a sizeable untagged asymmetry  $A_{\text{CP}}(B \rightarrow X_{s+d}\gamma) \sim A_{\text{CP}}(B \rightarrow X_s\gamma)$ . The only exception is the case of a cancellation between the two terms in (51), which is only possible for  $A_{\text{CP}}(B \rightarrow X_d\gamma) \sim 100\%$ . These considerations lead to the following general questions: (i) To which extent are the untagged CP asymmetry  $A_{\text{CP}}(B \rightarrow X_{s+d}\gamma)$  and the tagged CP asymmetry  $A_{\text{CP}}(B \rightarrow X_s\gamma)$  correlated? (ii) To which extent can the untagged CP asymmetry  $A_{\text{CP}}(B \rightarrow X_{s+d}\gamma)$  be sensitive to the CP asymmetry in the  $d$  sector  $A_{\text{CP}}(B \rightarrow X_d\gamma)$ ?

Clearly, predictions for the normalized CP asymmetries in  $\bar{B} \rightarrow X_{s/d}\gamma$  beyond the SM are rather model-dependent [37, 38]. For example, supersymmetric predictions depend strongly on the assumptions for the supersymmetry-breaking sector [37, 38]. However, especially for the untagged CP asymmetry, specific properties can be identified within general classes of models. In the following sections we analyze the above questions in various supersymmetric scenarios, namely in so-called minimal flavour violation models with and without additional sources of CP violation. Moreover, we study also general flavour violation models using a model-independent approach.

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<sup>4</sup>The analogous SM test within exclusive modes is rather limited, because U-spin breaking effects cannot be calculated in a model-independent way. Estimates [8, 35] lead to the conclusion that the U-spin breaking effects are possibly as large as the rate differences themselves.

## 4 CP asymmetries within minimal flavour violation

### 4.1 RG-invariant definition of MFV

In the analysis of FCNC processes beyond the SM, especially within supersymmetry, the additional assumption of minimal flavour violation (MFV) is often introduced. MFV is then loosely defined as: ‘all flavour changing interactions are completely determined by the CKM angles’. Especially in a renormalization-group equation (RGE) approach, the naive assumption of MFV is problematic, since it is not stable under radiative corrections and calls for a more precise concept. In ref. [10], a consistent definition was presented, which essentially also requires that all flavour and CP-violating interactions are linked to the known structure of Yukawa couplings. The constraint is introduced with the help of a symmetry concept and can be shown to be RGE invariant, which is a crucial ingredient for a consistent effective field theory approach.

In fact, it is well known that the maximal flavour symmetry group of unitary field transformations allowed by the gauge part of the SM Lagrangian,  $U(3)^5$ , can be decomposed in the following way

$$G_F \equiv SU(3)_q^3 \otimes SU(3)_\ell^2 \otimes U(1)_B \otimes U(1)_L \otimes U(1)_Y \otimes U(1)_{PQ} \otimes U(1)_{E_R} , \quad (54)$$

where

$$SU(3)_q^3 = SU(3)_{Q_L} \otimes SU(3)_{U_R} \otimes SU(3)_{D_R} , \quad (55)$$

$$SU(3)_\ell^2 = SU(3)_{L_L} \otimes SU(3)_{E_R} . \quad (56)$$

The subgroup  $SU(3)_q^3 \otimes SU(3)_\ell^2 \otimes U(1)_{PG} \otimes U(1)_{E_R}$  is broken by the Yukawa part of the SM. Nevertheless, one can formally promote the group  $G_F$  to an exact symmetry by assuming that the Yukawa matrices are vacuum expectation values of dimensionless auxiliary fields  $Y_U$ ,  $Y_D$ , and  $Y_E$  transforming under  $SU(3)_q^3 \otimes SU(3)_\ell^2$  as

$$Y_U \sim (3, \bar{3}, 1)_{SU(3)_q^3} , \quad Y_D \sim (3, 1, \bar{3})_{SU(3)_q^3} , \quad Y_E \sim (3, \bar{3})_{SU(3)_\ell^2} . \quad (57)$$

By definition, an effective theory satisfies the MFV criterion if all higher-dimensional operators, constructed from SM and  $Y$  fields, are invariant under CP and (formally) under the flavour group  $G_F$ . Thus, MFV requires the dynamics of flavour violation to be completely determined by the structure of the ordinary Yukawa couplings. This also means that all CP violation originates from the CKM phase [10]. We note here that one can extend this consistent concept of MFV by adding flavour-blind phases. In this case CP is not only broken by the CKM phase but also by these additional phases. However, the important property of renormalization-group invariance of the concept of MFV is untouched.

The hierarchical structure of the CKM matrix and of the Yukawa couplings restricts the number of relevant operators significantly. This leads to one of the key predictions of the MFV: the existence of a direct link between the  $b \rightarrow s$ ,  $b \rightarrow d$  and  $s \rightarrow d$  transitions. This prediction, within the  $\Delta F = 1$  sector, is definitely not well tested at the moment.

## 4.2 Flavour-blind supersymmetric models with and without extra phases

In supersymmetric theories a necessary condition for the fulfillment of the MFV requirement is that all soft SUSY-breaking terms can be diagonalized by superfield rotations. Note that this is a non-trivial statement, because the two matrices  $\tilde{V}_L$  and  $\tilde{V}_R$ , which diagonalize the soft SUSY-breaking masses squared for the left and right squarks, respectively, must also diagonalize the left-right mixing matrix. Provided that the above statement is correct, one can put all the information on flavour changing couplings inside the Yukawas of the superpotential

$$Y_U^{\text{diag}} = V_{CKM} D_L Y_U U_R^\dagger, \quad Y_D^{\text{diag}} = D_L Y_D D_R^\dagger. \quad (58)$$

A sufficient condition for the MFV concept to be realized is that  $D_L$ ,  $U_R$  and  $D_R$  are unit matrices. If we allow for additional phases, these are unit matrices times a phase.

We realize both options for MFV models, with and without additional phases, by a flavour-blind Minimal Supersymmetric standard model (MSSM) with conserved R-parity, where all the soft breaking terms are generated at the GUT scale and evolved to the electroweak scale by two-loop RGEs [39–41]. We define the soft breaking terms at the GUT scale as

$$\begin{aligned} (M_a^2)_{ij} &= M_A^2 \delta_{ij} \quad (a = Q, U, D, L, E) \\ (Y_a^A)_{ij} &= A_a e^{i\phi_{Aa}} (Y_a)_{ij} \quad (a = U, D, E) \\ M_{H_1}^2, M_{H_2}^2 \\ B e^{i\phi_B} \\ e^{i\phi_a} M_a \quad (a = 1, 2, 3) \end{aligned} \quad (59)$$

where  $i, j$  are family indices, the  $Y_f^A$  are trilinear scalar couplings and  $Y_f$  denote the Yukawa matrices;  $M_a^2$  are the soft SUSY breaking masses for the sfermions. In contrast to the analysis in [42], we do our analysis within the most general flavour blind analysis and do not assume any additional constraint on the soft breaking terms such as universality or  $SU(5)$  symmetry;  $M_{H_1}$  and  $M_{H_2}$  represent the Higgs soft breaking masses and  $B$  mixes both Higgs doublets. Beside the parameters  $A_a$ ,  $B$ ,  $M_a$  of Eq. (59) also  $\mu$  can be complex, yielding a total of six phases. Two of these phases can be eliminated because of a Peccei–Quinn symmetry and an R-symmetry [43]. We work in a basis where  $B$  and  $M_2$  are real. For simplicity we also assume that the remaining gaugino phases are real.

Our flavour-blind assumptions at the GUT scale (59) are compatible with the general MFV scenario. In fact, the two properties, namely that the soft contributions of the scalar mass are universal in generation space and that the trilinear soft terms are proportional to Yukawa couplings, are sufficient conditions for MFV. At an arbitrary scale, however, the physical squark masses are not equal, but the induced flavour violation is still described in terms of the usual CKM parameters. Having used the RG equation we arrive at the following mass terms for left sfermions:

$$(M_Q^2)_{ij} = M_{Q,0}^2 \times [\alpha_0^Q \delta_{ij} + \alpha_1^Q (Y_U Y_U^\dagger)_{ij} + \alpha_2^Q (Y_D Y_D^\dagger)_{ij} + \dots] \quad (60)$$

Owing to the hierarchical structure of the flavour parameters, the higher order terms in  $Y_a$  are numerically strongly suppressed. Analogous statements about the other sfermion mass parameters as well as the trilinear couplings are also valid.

As mentioned above we now consider two options in our analysis. In the first scenario, we put all flavour-blind phases in (59) to zero; thus, we are then in the strict MFV scenario, where the only source of CP violation is the CKM matrix. In the second scenario, we keep the phases for the  $A$ -parameters and for  $\mu$ . However, then we have to take into account the constraints of the electric dipole moments (EDM) of the electron and of the neutron. Contrary to the SM, where the EDMs occur at the higher loop only and the theoretical predictions are very small, the SUSY contributions appear already at one-loop order leading to theoretical predictions, which in general exceed the experimental bounds. The resulting strong constraints on the complex phases within SUSY models reflect the well-known SUSY-CP problem<sup>5</sup>. We will investigate how the EDM constraints restricts the ranges for the CP asymmetries in the  $b$  system.

### 4.3 Numerical analysis

In this section we present our numerical results for the CP asymmetries for the two scenarios discussed before. Before proceeding we briefly summarize the main procedure to calculate the parameters at the electroweak scale. The gauge couplings  $g_1, g_2, g_3$  and the Yukawa couplings are calculated in the  $\overline{DR}$  scheme by adopting the shifts given in [45]. In case of the top and bottom Yukawa couplings, we include the two-loop gluonic part [46] in the shifts. In the case of the bottom and tau Yukawa couplings, we resum the SUSY contributions as proposed in [47–49].

These parameters are evolved to  $M_{\text{GUT}}$  using two-loop RGEs [39–41]. At two-loop order the gauge couplings do not meet exactly [50, 51]; the differences are due to threshold effects at the unification scale  $M_{\text{GUT}}$  and leave us with an ambiguity in the definition of  $M_{\text{GUT}}$ . In this paper we define  $M_{\text{GUT}}$  as the scale where  $g_1 = g_2$  in the RGE evolution. At the scale  $M_{\text{GUT}}$  the boundary conditions for the soft SUSY-breaking parameters are imposed. All parameters are evolved to  $M_{\text{SUSY}} \equiv \sqrt{\overline{m}_{\tilde{t}_1} \overline{m}_{\tilde{t}_2}}$  using two-loop RGEs. At this scale the masses of all SUSY particles are calculated using one-loop formulae (which are a three-generation extension of those presented in [45]). In the case of the masses of the neutral Higgs bosons two-loop contributions as given in [52, 53] are included. The absolute value of  $\mu$  is, as usual, obtained from radiative electroweak symmetry breaking while its phase is completely arbitrary. Here we have included the complete one-loop contributions [45] as well as the leading two-loop contributions given in [53]. The complete procedure is iterated until the resulting masses change by less than one per-mill between two iterations. For further technical details on the procedure see [54].

Once a stable solution has been found, the couplings are evolved from  $M_{\text{SUSY}}$  to  $m_t(m_t)$ , where the contributions to the Wilson coefficients are calculated and then to  $m_W$  where the

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<sup>5</sup>Note, that the constraints for the electron EDM are less severe once additional phases for flavour violating parameters are taken into account [44].



EDMS are calculated. In the second scenario with complex phases, we take into account the constraints due to the bounds on the neutron EDM and the electron EDM:

$$|d_n|_{\text{exp}} \leq 6 \times 10^{-26} e \text{ cm}, \quad |d_e|_{\text{exp}} \leq 7 \times 10^{-28} e \text{ cm}. \quad (61)$$

The calculation of the neutron EDM is performed in two different neutron models, the Chiral Quark model and the Quark–Parton model, to get an estimate of the involved theoretical uncertainty. Here we have used the formulae for both models as presented in [55]. We select the points within the supersymmetric parameter space, which are compatible with the constraint on the neutron EDM in the following way: the phases in the trilinear terms,  $\phi_{AU}$  and  $\phi_{AD}$ , (see Eq. (59)) are randomly chosen, and then the phase of the parameter  $\mu$  is chosen such that the experimental constraint on the neutron EDM is fulfilled for at least one of the two neutron models. Finally, the experimental bound on the electron EDM, induced by the phase of  $\mu$  and  $\phi_{AE}$  can be fulfilled by an appropriate choice of the latter phase.

For the numerical results presented below we have varied the parameters in the following ranges:

$$\tan \beta \in [2, 50] \quad (62)$$

$$M_{1/2} \in [100, 1000] \text{ GeV} \quad (63)$$

$$M_{Hi}, M_a \in [100, 1000] \text{ GeV} \quad (a = Q, U, D, L, E, i = 1, 2) \quad (64)$$

$$|A_u| \leq \sqrt{3(M_Q^2 + M_U^2 + M_{H_2}^2)} \quad (65)$$

$$|A_d| \leq \sqrt{3(M_Q^2 + M_D^2 + M_{H_1}^2)} \quad (66)$$

$$|A_e| \leq \sqrt{3(M_L^2 + M_E^2 + M_{H_1}^2)} \quad (67)$$

The range of the  $A$  parameters is restricted to avoid the danger of colour and/or charge breaking minima.

Scanning the parameter space of the MFV scenario without extra flavour-blind phases as described above, we find that the untagged CP asymmetry is completely unaffected. From Fig. 1 we see that there are only tiny effects at the 0.02% level. This shows the stability of the strict MFV concept in running from the GUT scale down to the electroweak scale, via fourteen orders of magnitude, due to the hierarchical structure of the flavour parameters. We also note that the tagged  $b \rightarrow s$  CP asymmetry allows for a bigger ( $\sim \pm 0.2\%$ ) but still unobservable deviation from the SM value.

Allowing for extra flavour-blind phases for the  $A$ - and  $\mu$  parameters at the GUT scale, significant larger effects in the untagged asymmetry are possible, as can be seen in Fig. 2 where we present the results of the parameter scanning. In all the plots, the black (green) points have been obtained with (without) requiring the EDMs constraint. In Fig. 2a we plot the untagged asymmetry as a function of  $B(b \rightarrow s\gamma)$ . One clearly sees that possible effects are still much below the 5% threshold, in particular if the EDM constraint is imposed. Therefore, a clear discrimination between minimal and general flavour models is still possible via the untagged CP asymmetry as we will explore more concretely in the next section. In Fig. 2b, we show the strong correlation between the untagged and the tagged  $b \rightarrow s$  CP

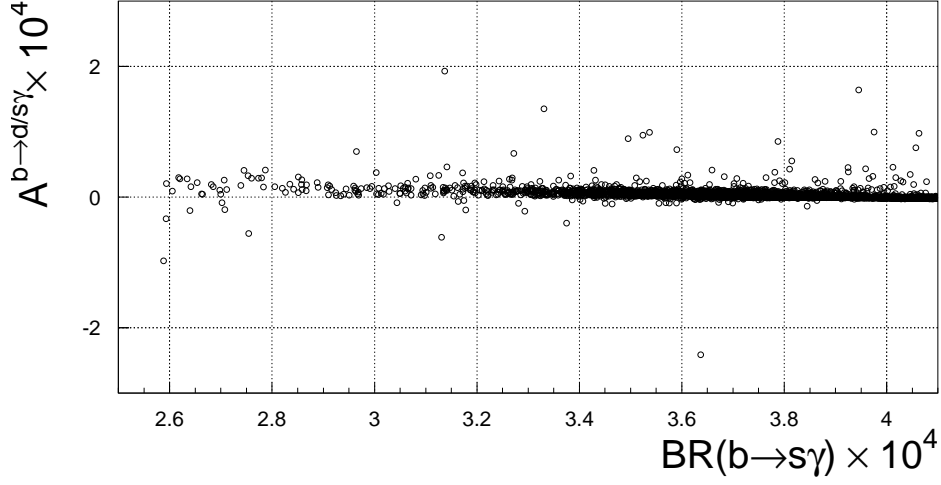


Figure 1: Untagged CP asymmetry in the MFV scenario without flavour-blind phases.

asymmetry. Supersymmetric contributions to the magnetic and chromo-magnetic Wilson coefficients are complex, but identical for the  $s$  and  $d$  sectors:  $C_7^d = C_7^s$  and  $C_8^d = C_8^s$  to a very high precision. These relations then imply a strong correlation between the new physics contributions to the normalized CP asymmetries. They also imply that the ratio  $R_{ds}$  does not deviate appreciably from its SM value:  $R_{ds} \approx R_{ds}^{SM}$ . This implies linear proportionality between tagged and untagged CP asymmetries:

$$A_{CP}(B \rightarrow X_{d/s}\gamma)_{\text{flavourblind}} \sim \frac{A_{CP}(B \rightarrow X_s\gamma)}{1 + R_{ds}} \sim A_{CP}(B \rightarrow X_s\gamma) . \quad (68)$$

The scatter plot in Fig. 2c shows no direct correlation of the untagged CP asymmetry to the tagged one in the  $b \rightarrow d$  mode.

We conclude that, with respect to a minimal flavour-violating scenario, there is no theoretical reason to make an extra effort to measure the tagged CP asymmetry in the  $b \rightarrow s$  mode, if the untagged CP asymmetry is measured: the untagged measurement represents the cleaner test of the SM. We also stress that a very high precision would be needed in order to separate MFV scenarios with and without flavour-blind phases by a measurement of the untagged CP asymmetry.

## 5 Model-independent analysis of CP asymmetries

The model-independent formulae of the CP asymmetries Eqs. (42) and (43), based on the operator basis of Eq. (6), allow us to analyze new physics scenarios considerably more general than the minimal flavour-violating MSSM. Taking into account the experimental bounds on

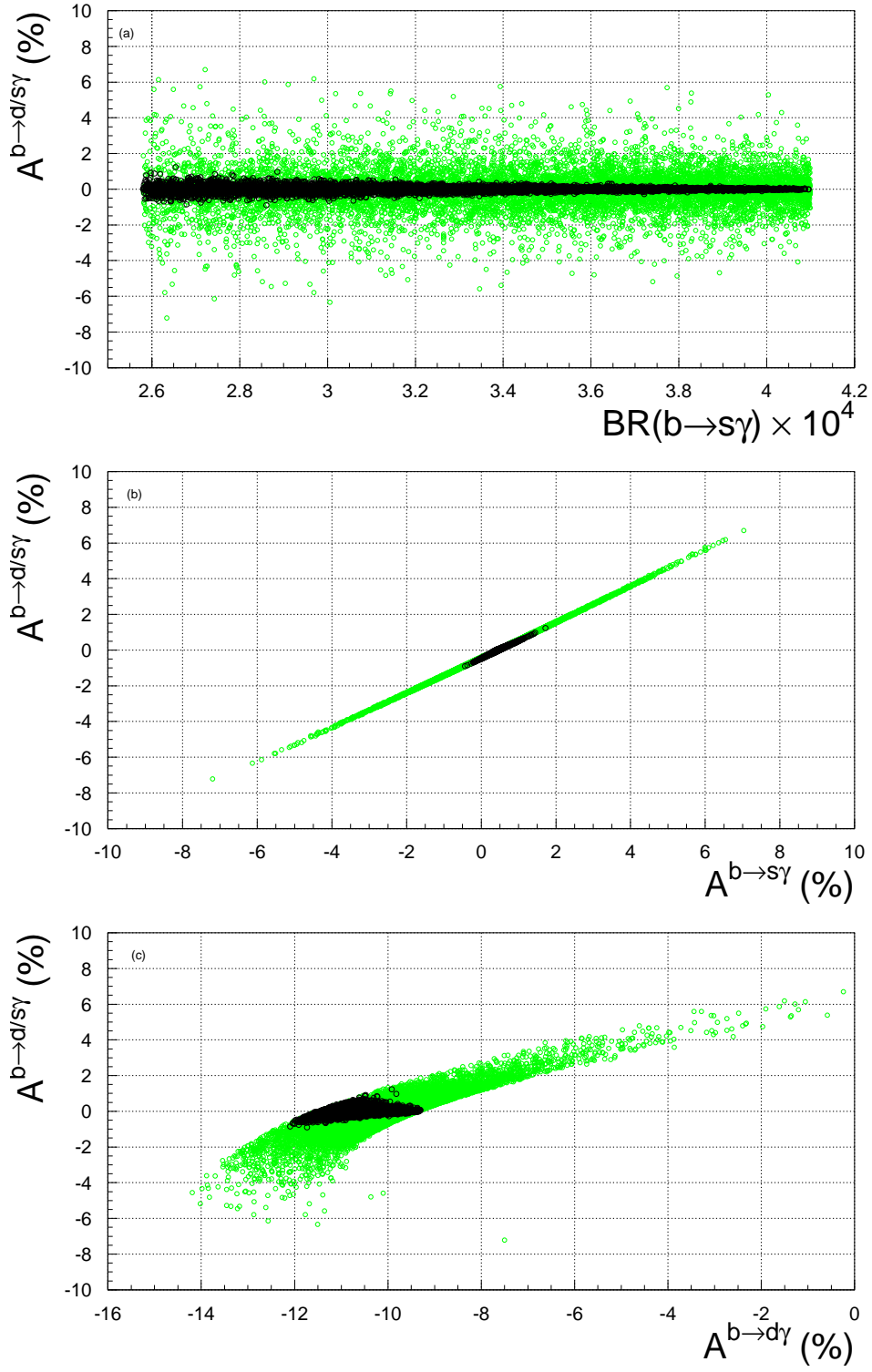


Figure 2: Untagged rate asymmetry in the MFV scenario with non-vanishing flavour-blind phases. The EDM constraint is relaxed for the green points and imposed on the black ones.

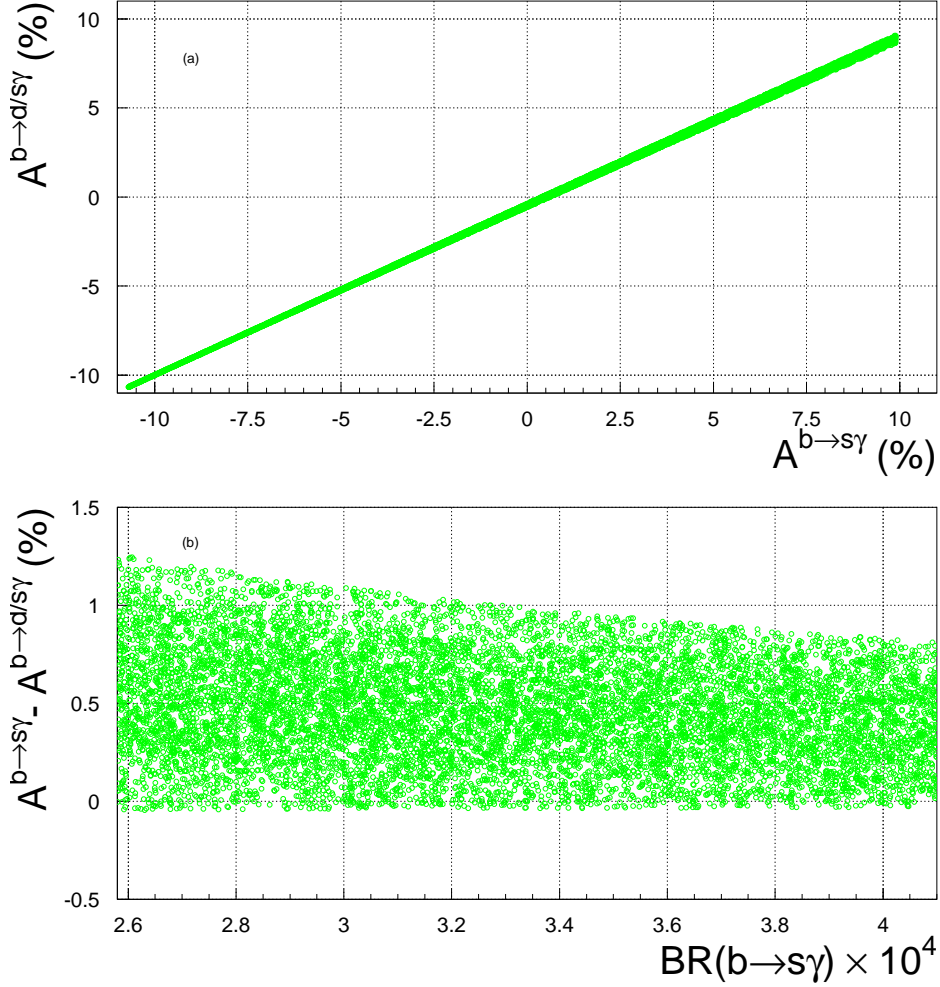


Figure 3: Model-independent analysis with new physics in  $C_{7,8}^s$ . Correlation between the tagged and untagged CP asymmetries.

the  $\bar{B} \rightarrow X_s \gamma$  branching ratio and CP asymmetry given in Eqs. (1) and (2), we investigate the untagged CP asymmetry. In particular, we analyze possible correlations between the tagged and the untagged measurements and their indirect sensitivity to the CP asymmetry in the  $\bar{B} \rightarrow X_d \gamma$  mode, which will not be directly measurable in the near future.

Within this model-independent analysis, the  $b \rightarrow s$  or  $b \rightarrow d$  sectors are uncorrelated and described by the Wilson coefficients  $C_{7,8}^s$  and  $C_{7,8}^d$ , respectively. In the following we study two distinct scenarios in which either  $C_{7,8}^s$  or  $C_{7,8}^d$  are allowed to differ from their SM values. These scenarios are very different from any minimal flavour-violating model (not necessarily within a supersymmetric framework) in which the  $d$  and  $s$  sectors are always correlated:  $C_{7,8}^s \stackrel{\text{MFV}}{=} C_{7,8}^d$ .

We summarize the results of the analysis of the scenario with new physics in  $C_{7,8}^s$  in the

scatter plots presented in Fig. 3. The points are generated varying  $C_{7,8}^s(\mu_0)$  in the complex plane and imposing the experimental constraints  $\bar{B} \rightarrow X_s \gamma$  and  $\bar{B} \rightarrow X_s g$ . The latter, in particular, provides the following loose constraint onto  $C_8^s$  [56]:  $|C_8^s(\mu_0)/C_8^{s,\text{SM}}(\mu_0)| < 10$ .

In this scenario, the  $\bar{B} \rightarrow X_s \gamma$  CP asymmetry receives large contributions and, indeed, it saturates the experimental bound given in Eq. (3). Since the  $b \rightarrow d$  sector is unaffected, we expect a strict proportionality between the tagged and untagged CP asymmetries, as can be seen from Fig. 3a. In Fig. 3b figure, we show in detail the difference between the tagged and untagged CP asymmetries as a function of the  $\bar{B} \rightarrow X_s \gamma$  branching ratio. We see that the difference between the two asymmetries is always below 1.3%. Note, finally, that the difference between the two CP asymmetries is always positive; in fact,

$$A_{\text{CP}}^s - A_{\text{CP}}^{d/s} = \frac{A_{\text{CP}}^s(1 - R_{ds}) - A_{\text{CP}}^d R_{ds}}{1 + R_{ds}} \simeq -\frac{R_{ds}}{1 + R_{ds}} A_{\text{CP}}^{d,\text{SM}} > 0. \quad (69)$$

In the second scenario, new physics is present only in the  $d$  sector and we are able to explore the sensitivity of the untagged CP asymmetry to possible novel effects in the  $b \rightarrow d$  mode. From Fig. 4, it is clear that such sensitivity is very restricted; even for very large new physics effects in the  $b \rightarrow d$  CP asymmetry or branching ratio, the untagged asymmetry does not exceed values of 2%. In each figure, the shaded band represents the SM predictions for  $A_{\text{CP}}(B \rightarrow X_d \gamma)$  and for the ratio  $R_{ds}$ . Figure 4a shows the correlation between branching ratio and CP asymmetry in  $\bar{B} \rightarrow X_d \gamma$ . In Figs. 4b and 4c, we illustrate the correlation between the  $B \rightarrow X_d \gamma$  branching ratio and the untagged CP asymmetry and between the tagged and the untagged CP asymmetry respectively.

We cross-checked our model-independent analysis in a general flavour violation scenario within supersymmetry, using the mass insertion method. The mass insertion approximation is a well-known useful tool to study the effect of the large number of flavour-changing parameters present in the MSSM. The idea is to move into a basis in which the Yukawas are diagonal without introducing relative rotations between particles and the corresponding superpartners (i.e. we consider rigid superfield transformations). In this so-called super-CKM basis, the squark mass matrices are non-diagonal and represent generic new sources of flavour-changing neutral currents within the MSSM. We can then expand the physical amplitudes in powers of these off-diagonal elements, assuming that they are small with respect to the diagonal entries. A given process is thus dominated by only few of these mass insertions. A comprehensive analysis of all the insertions for the CP asymmetries in the spirit of the analysis [57, 58]) is beyond the scope of the present analysis. However, we analyzed gluino contributions with non-vanishing  $(m_{LR}^2)_{23}^d$  or  $(m_{LR}^2)_{13}^d$  and their chiral analogue. We always consider mass insertions normalized to the average down squarks mass, i.e.

$$(\delta_{LR}^d)_{23} = \frac{(m_{LR}^2)_{23}^d}{\tilde{m}^2} \quad \text{and} \quad (\delta_{LR}^d)_{13} = \frac{(m_{LR}^2)_{13}^d}{\tilde{m}^2}, \quad (70)$$

where we choose  $\tilde{m} \sim 500 \text{ GeV}$ . Moreover, we have taken into account bounds on the mass insertions induced by all the various experimental constraints (see Ref. [59] for a detailed description). In the two separate scenarios in which either  $(\delta_{LR}^d)_{23} \neq 0$  or  $(\delta_{LR}^d)_{13} \neq 0$ , we have not found any significant deviation from the model-independent results presented above.

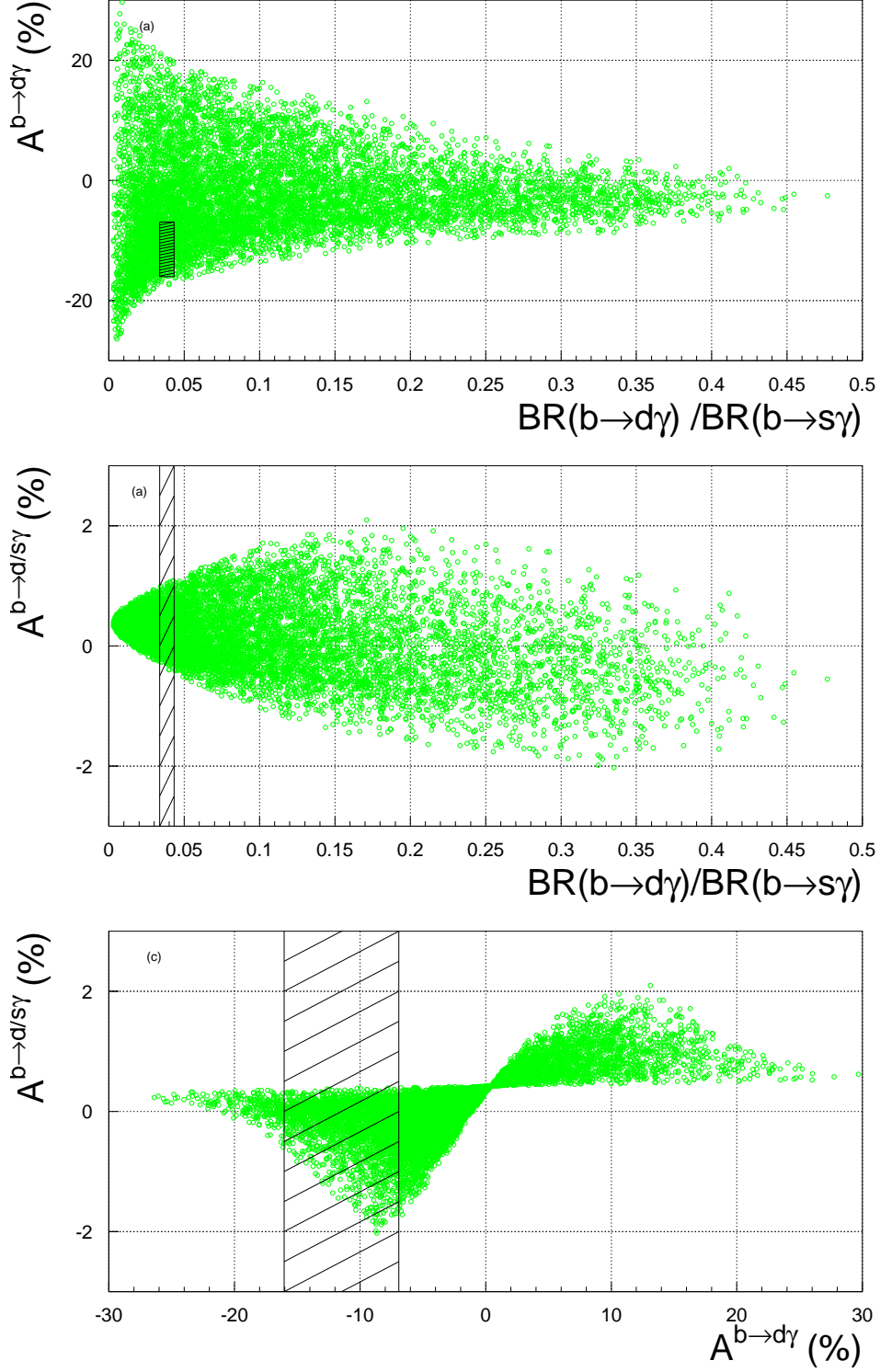


Figure 4: Model-independent analysis with new physics in  $C_{7,8}^d$ . Correlation between the branching ratio and CP asymmetry in  $B \rightarrow X_d \gamma$  and the untagged CP asymmetry. The shaded areas corresponds to the SM prediction.

## 6 Conclusions

We have presented updated SM predictions for the branching ratios  $\bar{B} \rightarrow X_q \gamma$  ( $q = s, d$ ) and the corresponding CP asymmetries together with model-independent formulae that can be used to study the impact of generic new physics interactions on these observables. We have shown that the untagged CP asymmetry, i.e. the CP asymmetry in the  $\bar{B} \rightarrow X_{s+d} \gamma$  mode, is extremely sensitive to new physics contributions. In the SM, in fact, this observable is negligibly small thanks to U-spin relations and to the unitarity of the CKM matrix and allows for a clean test, whether additional CP phases are present or not.

Using the model-independent formulae, we have analysed the untagged CP asymmetry in several scenarios beyond the SM. We considered the MSSM with minimal flavour violation and a model with generic contributions to the Wilson coefficients  $C_7$  and  $C_8$ .

MFV models are characterized by the requirement of expressing all flavour-changing interactions in terms of powers of the Yukawa matrices. In a first stage, we assumed the CKM phase to be the only CP phase present at the grand unification scale. In this restricted scenario we find that the untagged CP asymmetry receives only very small contributions: this class of models cannot be distinguished from the SM with the help of this observable. Subsequently, we allowed the  $\mu$  and  $A$  parameters to be complex. After the EDM bounds are taken into account, only asymmetries below the 2% level survive and we find a strict proportionality between the untagged ( $\bar{B} \rightarrow X_{s+d} \gamma$ ) and tagged ( $\bar{B} \rightarrow X_s \gamma$ ) CP asymmetries. The task of distinguishing these two MFV scenarios is beyond the possibilities of the existing  $B$ -factories but should be within the reach of future experiments.

In the model-independent approach, we have allowed for new physics contributions to the  $s$  and  $d$  sectors independently. In the first case, the untagged CP asymmetry can be as large as  $\pm 10\%$ , once the recent experimental data from Belle on the CP asymmetries are taken into account; more importantly, we found that the tagged and untagged asymmetries are again strictly proportional to each other. In the second case with new physics in the  $d$  sector, we have not found untagged CP asymmetries larger than 2%: this implies that the untagged CP asymmetry is not really sensitive for new physics effects in the  $d$  sector.

With the expected experimental accuracy of  $\pm 3\%$  at the  $B$ -factories, a clear distinction between a minimal and a more general flavour model is possible through the untagged CP asymmetry.

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# Appendix

We collect the explicit expressions for many of the quantities introduced in sect. 2:

- The ratio of the  $\overline{\text{MS}}$  running mass of the bottom quark ( $m_b^{\overline{\text{MS}}}(\mu_0)$ ) to the 1S mass ( $m_b^{1S}$ ) is [21] ( $\mu_0 = m_t$ ):

$$r(\mu_0) = 0.578 \left( \frac{\alpha_s(M_Z)}{0.1185} \right)^{-1.0} \left( \frac{m_b^{1S}}{4.69} \right)^{0.23} \left( \frac{m_c(m_c)}{1.25} \right)^{-0.003} \left( \frac{\mu_0}{165} \right)^{-0.08} \left( \frac{\mu_b}{4.69} \right)^{0.006}. \quad (71)$$

- The charm contribution of the perturbative part is given by :

$$K_c^{(0)} = \sum_{k=1}^8 \eta^{a_k} d_k, \quad (72)$$

$$K_c^{(11)} = \frac{\alpha_s(\mu_b)}{4\pi} \sum_{k=1}^8 \eta^{a_k} \left[ 2\beta_0 a_k d_k \left( \log \frac{m_b}{\mu_b} + \eta \log \frac{\mu_0}{m_W} \right) + \tilde{d}_k + \tilde{d}_k^\eta \eta \right. \\ \left. + \text{Re} \left[ \tilde{d}_k^a a(z) + \tilde{d}_k^b b(z) \right] \right], \quad (73)$$

$$K_c^{(12)} = \frac{\alpha_s(\mu_b)}{4\pi} \sum_{k=1}^8 \eta^{a_k} \left[ \tilde{d}_k^{i\pi} \pi + \text{Im} \left[ \tilde{d}_k^a a(z) + \tilde{d}_k^b b(z) \right] \right], \quad (74)$$

$$K_c^{(13)} = \frac{\alpha_s(\mu_b)}{4\pi} \sum_{k=1}^8 \eta^{a_k} \text{Re} \left[ \tilde{d}_k^a a(z) + \tilde{d}_k^b b(z) \right], \quad (75)$$

$$K_c^{(14)} = \frac{\alpha_s(\mu_b)}{4\pi} \sum_{k=1}^8 \eta^{a_k} \text{Im} \left[ \tilde{d}_k^a a(z) + \tilde{d}_k^b b(z) \right], \quad (76)$$

where  $\eta = \alpha_s(\mu_0)/\alpha_s(\mu_b)$ ,  $\beta_0 = 23/3$ ,  $z = (m_c/m_b)^2$ ; the magic numbers  $a_k$ ,  $d_k$ ,  $\tilde{d}_k$ ,  $\tilde{d}_k^a$ ,  $\tilde{d}_k^b$ ,  $\tilde{d}_k^{i\pi}$  can be found in Table 2 of Ref. [32] and the functions  $a(z)$ ,  $b(z)$  are presented in Appendix D of Ref. [21].

- The leading order top contribution is:

$$K_t^{(01)} = \eta^{\frac{4}{23}} \frac{23}{36} - \frac{8}{3} (\eta^{\frac{4}{23}} - \eta^{\frac{2}{23}}) \frac{1}{3}, \quad (77)$$

$$K_t^{(02)} = \eta^{\frac{4}{23}}, \quad (78)$$

$$K_t^{(03)} = -\frac{8}{3} (\eta^{\frac{4}{23}} - \eta^{\frac{2}{23}}). \quad (79)$$

- The next-to-leading order top contribution is:

$$K_t^{(11)} = -\frac{2}{9} \alpha_s(m_b)^2 \left( \eta^{\frac{4}{23}} \frac{23}{36} - \frac{8}{3} (\eta^{\frac{4}{23}} - \eta^{\frac{2}{23}}) \frac{1}{3} \right) \\ + \frac{\alpha_s(\mu_0)}{\pi} \log \frac{\mu_0}{m_t} 4x \frac{\partial}{\partial x} \left[ -\frac{1}{2} \eta^{\frac{4}{23}} A_0^t(x_t) + \frac{4}{3} (\eta^{\frac{4}{23}} - \eta^{\frac{2}{23}}) F_0^t(x_t) \right]$$



$$\begin{aligned}
& + \frac{\alpha_s(\mu_b)}{4\pi} \left\{ E_0^t(x_t) \sum_{k=1}^8 e_k \eta^{a_k + \frac{11}{23}} - 2\eta^{\frac{4}{23}} \left[ \frac{1}{4} \eta A_1^t(x_t) + \left( \frac{12523}{3174} - \frac{7411}{4761} \eta - \frac{2}{9} \pi^2 \right. \right. \right. \\
& - \frac{4}{3} \left( \log \frac{m_b}{\mu_b} + \eta \log \frac{\mu_0}{m_t} \right) \left. \left. \left. \right) \frac{23}{36} - \frac{2}{3} \eta F_1^t(x_t) + \left( -\frac{50092}{4761} + \frac{1110842}{357075} \eta \right. \right. \right. \\
& + \frac{16}{27} \pi^2 + \frac{32}{9} \left( \log \frac{m_b}{\mu_b} + \eta \log \frac{\mu_0}{m_t} \right) \left. \left. \left. \right) \frac{1}{3} \right] - 2\eta^{\frac{2}{23}} \left[ \frac{2}{3} \eta F_1^t(x_t) \right. \right. \\
& \left. \left. \left. + \left( \frac{2745458}{357075} - \frac{38890}{14283} \eta - \frac{4}{9} \pi^2 - \frac{16}{9} \left( \log \frac{m_b}{\mu_b} + \eta \log \frac{\mu_0}{m_t} \right) \right) \frac{1}{3} \right] \right\}, \quad (80)
\end{aligned}$$

$$\begin{aligned}
K_t^{(12)} &= -\frac{2}{9} \alpha_s(m_b)^2 \eta^{\frac{4}{23}} \\
& - \frac{\alpha_s(\mu_b)}{2\pi} \eta^{\frac{4}{23}} \left( \frac{12523}{3174} - \frac{7411}{4761} \eta - \frac{2}{9} \pi^2 - \frac{4}{3} \left( \log \frac{m_b}{\mu_b} + \eta \log \frac{\mu_0}{m_t} \right) \right), \quad (81)
\end{aligned}$$

$$\begin{aligned}
K_t^{(13)} &= \frac{16}{27} \alpha_s(m_b)^2 (\eta^{\frac{4}{23}} - \eta^{\frac{2}{23}}) - \frac{\alpha_s(\mu_b)}{2\pi} \left\{ \right. \\
& \eta^{\frac{4}{23}} \left( -\frac{50092}{4761} + \frac{1110842}{357075} \eta + \frac{16}{27} \pi^2 + \frac{32}{9} \left( \log \frac{m_b}{\mu_b} + \eta \log \frac{\mu_0}{m_t} \right) \right) \\
& \left. + \eta^{\frac{2}{23}} \left( \frac{2745458}{357075} - \frac{38890}{14283} \eta - \frac{4}{9} \pi^2 - \frac{16}{9} \left( \log \frac{m_b}{\mu_b} + \eta \log \frac{\mu_0}{m_t} \right) \right) \right\}, \quad (82)
\end{aligned}$$

$$K_t^{(14)} = \frac{2\alpha_s(\mu_b)}{27} \eta^{\frac{2}{23}}, \quad (83)$$

$$K_t^{(15)} = \frac{2\alpha_s(\mu_b)}{9} \eta^{\frac{2}{23}}, \quad (84)$$

where the numbers  $e_k$  and the functions  $A_1^t$  and  $F_1^t$  are given in Ref. [21].

- The electroweak contributions are [60–63]:

$$\varepsilon^{(11)} = \frac{\alpha_{\text{em}}(m_Z)}{\alpha_s(\mu_b)} \left( \frac{88}{575} \eta^{\frac{16}{23}} - \frac{40}{69} \eta^{-\frac{7}{23}} + \frac{32}{75} \eta^{-\frac{9}{23}} \right) - \frac{\alpha_{\text{em}}(m_Z)}{\pi} \log \frac{m_Z}{\mu_b} r(\mu_0) \eta^{\frac{4}{23}}, \quad (85)$$

$$\begin{aligned}
\varepsilon^{(12)} &= \frac{\alpha_{\text{em}}(m_Z)}{\alpha_s(\mu_b)} \left( -\frac{704}{1725} \eta^{\frac{16}{23}} + \frac{640}{1449} \eta^{\frac{14}{23}} + \frac{32}{1449} \eta^{-\frac{7}{23}} - \frac{32}{575} \eta^{-\frac{9}{23}} \right) \\
& - \frac{\alpha_{\text{em}}(m_Z)}{\pi} \log \frac{m_Z}{\mu_b} r(\mu_0) \frac{8}{3} (\eta^{\frac{2}{23}} - \eta^{\frac{4}{23}}). \quad (86)
\end{aligned}$$

- Finally, the bremsstrahlung functions  $\phi(\delta, z)$  appearing in the expression for  $B(E_0)$  in Eq. (28) coincide with the functions given in Appendix E of Ref. [21] with the only exception of  $\phi_{27}$  (and consequently also of  $\phi_{17}$ ,  $\phi_{28}$  and  $\phi_{18}$ ) which has to be replaced by

$$\phi_{27}(\delta) = -\frac{8z^2}{9} \left[ \delta \int_0^{(1-\delta)/z} dt \left( G(t) + \frac{t}{2} \right) + \int_{(1-\delta)/z}^{1/z} dt (1-zt) \left( G(t) + \frac{t}{2} \right) \right], \quad (87)$$

where  $G(t)$  is given in Eq. (E.8) of Ref. [21].

- The functions in Eq. (39) are given by

$$\Delta\Gamma_{qg\gamma} = \frac{2\alpha_s(\mu_b)}{\pi} \left( C_2(\mu_b) - \frac{1}{6}C_1(\mu_b) \right) \text{Im}[\phi_{27}(\delta)] \text{Im} \left[ \left( C_7(\mu_b) - \frac{1}{3}C_8(\mu_b) \right) (1 + \epsilon_q^*) \right] \quad (88)$$

$$\Delta\Gamma_{q\gamma} = 2 \text{Im} \left[ -K_c^{(0)} K_c^{(14)} \epsilon_q + r(\mu_0) \left( K_t^{(0)} K_t^{(1)i*} + K_c^{(14)} K_t^{(0)} \epsilon_q^* + K_c^{(12)} K_t^{(0)} - K_c^{(0)} K_t^{(1)i} \right) \right]. \quad (89)$$

In comparing with the results given in the first reference in [37], we note that  $zb(z, \delta) = 9/(8\pi)\text{Im}[\phi_{27}(\delta)]$ .

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